Report on the State of Research in the 5-Dimensional Projective Unified Field Theory

Ernst Schmutzer

Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, D-07743 Jena, Germany

Reprint requests to E. S.; E-mail: eschmu@aol.com

Z. Naturforsch. **64a,** 273 – 288 (2009); received February 9, 2009

The author presents a historical sketch of the projective relativity theory before and (with new qualitative arguments) after World War II. Then he treats the development of his Projective Unified Field Theory since 1957 up till now with applications to a closed cosmological model, with the result of a vanishing big bang and satisfying numerical cosmological parameters in good agreement with the experiments.

Key words: 5-Dimensional Projective Relativity; No Big Bang; Cosmological Parameters.

1. Research on the Geometrical Program of a Unified Field Theory of Physics up to the World War II

1.1. Historical Annotations

It is well known, that Newtonian mechanics (inclusive Newton's gravitational theory) in the first centuries of its existence was very successful in terrestrial and planetary physics, and later even up to the distances of stars in our galaxy and also in more remote galaxies. Nevertheless, already in 1826 H. Olbers found discrepancies in applying this theory to cosmology. Further in 1859 U. Leverrier, in the course of evaluating a lot of empirical material, discovered the perihelion motion (slow rotation of the ellipse) of Mercury of about $43''$ per century, which could not be explained on the basis of the excellent Newtonian theory. Einstein's General Relativity Theory (1915), including his proper gravitational theory, was fully successful in explaining the perihelion motion and two further general-relativistic effects: frequency shift of photons and deflection of light in an external gravitational field. These three general-relativistic effects are known under the name "Einstein effects".

The fundament of the Einstein theory just mentioned is the concept of a 4-dimensional curved space-time with Riemannian geometry. This position opened the understanding of gravitation, not as an external Newtonian field in an absolute space-time, but as a geometrical property of the curved space-time. The "geometrization" of gravitation started its entry into physics.

Let me mention that the idea of the 4-dimensionality of space-time was borne after the development of the Special Relativity Theory (1905), but in this case for the still flat (non-curved) space-time, finally geometrically formulated by H. Minkowski (1908).

Einstein's geometrical gravitational theory won only very slowly recognition in the community of physicists. But nevertheless these ideas inspired the empirical investigation of the new Einstein effects mentioned.

1.2. Kaluza-Klein Approach

In the following years Einstein and other theoreticians were thinking about an amplification of the geometrization of a part of physics, particularly of the geometrization of the Maxwell theory of electromagnetic field, beside gravitation the only further field, well known and well-tried in physics at those days. Thus Einstein's program of a unified field theory of gravitation and electromagnetism was borne. But the empirical investigation of the Einstein effects which could step by step at least qualitatively be proved, encouraged the research in this field. An active impetus resulted primarily from the side of geometrical mathematicians. The great success in geometrizing gravitation inspired particularly Th. Kaluza (1921) to study the idea of geometrization for the case of electromagnetism. His basic idea was: maintaining Riemannian geometry as in the Einstein theory, but increasing the number of dimensions from four to five, i. e. to start with a 5-dimensional geometrical manifold and decompose it into the 4-dimensional space-time and a fifth 1-dimensional part. The calculations were rather lengthy and without an acceptable physical interpretation [1].

The first simplification of the calculations was reached by the cylindricity condition: independence of the 15 (because of symmetry properties) occurring 5-dimensional field functions on the 5$^{\text{th}}$ coordinate to reduce the field functions to 4-dimensional functions, i. e. to remain by means of physical arguments in the space-time.

The second simplification was the normalization condition: postulating constancy of an important 5-dimensional field function (e. g. $g_{55} = 1$). This way the number of the 5-dimensional field functions was reduced. Here the physical argument played an important role, namely that the number of the remaining 5-dimensional field functions is large enough for a unified field theory of gravitation and electromagnetism, which was the goal intended. Up to this time no further physical phenomena to be geometrized were known.

Following some years later Kaluza and the physicist O. Klein (1926) tried to find deeper physics in this direction, but combined with quantum mechanics. During the then following years this so-called Kaluza-Klein-formalism was formally improved, but it remained on the basis mentioned above without physical success.

1.3. Non-Symmetric Unified Field Theories

Parallel to these 5-dimensional attempts to a unified field theory of physics (gravitation and electromagnetism), Einstein continued with some co-workers, still in Berlin and then since 1933 in Princeton (USA) until his death 1955, his very intensive research with good hope, on following physico-geometrical subjects:

- As basis a 4-dimensional space-time (no change of the number of dimension).
- Instead of the Riemannian geometry of the General Relativity Theory choice of other types of higher geometries. Here two different directions of research played important roles:
  1. Non-symmetric metrical tensor instead of the usual symmetric tensor in order to amplify the number of field functions for grasping electromagnetism.
  2. Non-symmetric affinity (connection) in the definition of the covariant derivative (generalization of the partial derivative) of tensors, important for the transport of vectors in spaces with curvature. This idea opens the door to spaces with torsion (beside curvature).

For a period of about 30 years Einstein tested both variants. He preferred the second version, where the mathematics grew more and more complicated, but without accepted success. Only a very small group of co-workers was left, primarily led by P. G. Bergmann. As he told me in several private talks, Einstein was not willing to change to 5-dimensionality. The new situation after World War II will be treated later.

1.4. 5-Dimensional Projective Relativity Theories

In order to simplify my report, as usual I apply following conventions: X^μ are 5-dimensional homogeneous coordinates, x^i are 4-dimensional space-time coordinates. Greek indices run from 1 to 5, Latin indices from 1 to 4. The signature of space-time is $\{1, 1, 1, -1\}$. Comma denotes the partial derivative and semicolon the covariant derivative.

About one decade after Kaluza's step to the 5-dimensionality (with his heavy-going formalism), in my opinion a true mathematical break-through was reached by the geometry-mathematitians O. Veblen and B. Hoffmann (1931) as well as J. A. Schouten and D. van Dantzig (1932). They invented the new mathematical tool of the projectors, representing homogeneity properties of the 5-dimensional field functions. The following example of the homogeneity condition of a function $f(X^\mu)$ with the homogeneity degree a gives impression of this kind of tool:

$$f_{,\mu} X^\mu = af. \qquad (1)$$

Using this mathematical projector concept, for some years W. Pauli [2] in voluminous papers intensively investigated the path from this mathematical projective-relativistic framework to the true physical content of this scheme offered. His final decision with respect to the 5-dimensional theories was negative; therefore he left this direction of research and returned to the theory of elementary particles. Here his negative discussion with W. Heisenberg who tried to solve the problem of elementary particles by his non-linear spinor theory ("world formula") is also well known (1957).

2. Sketch of the Research on Non-Quantized Unified Field Theories after World War II

2.1. Revival of Projective Relativity Research by Jordan

As it is well known, P. Jordan is one of the fathers of quantum field theory (anticommutator quantization of the fermion field together with E. Wigner 1928). Later he changed from quantum theory to classical unified field theories, hoping that this way some basically open questions of the natural sciences could be solved:

- Explanation of the hypothesis of A. Wegener on the drift of the continents of the Earth.
- Explanation of suggested time-dependence of the Newtonian gravitational constant, idea induced by Dirac's hypothesis (1938) on the extremely large numbers in the existing Universe, etc.

Jordan's Main Ideas in Direction of the Unified Field Theories (1945)

First, for simplification using the 5-dimensional projective-relativistic mathematics. Second, abandoning the above mentioned normalization condition. This way a scalar field, till now having been treated as a constant, represented a new field function which could perhaps be useful for physics.

Here I should inform on Jordan's deeper insight into the 5-dimensional homogeneity mathematics mentioned with respect to possible application in physics: He recognized by group-theoretical investigations that the group of the 5-dimensional coordinate transformations of homogeneity degree 1 is equivalent to the semi-direct product of the group of the 4-dimensional coordinate transformations (typical for general-relativistic gravitation) and the group of the 4-dimensional electromagnetic gauge transformations (typical for electromagnetism) [3]. This knowledge was also an important argument for my own start of research on unified field theories, particularly for my preference of the projective-relativistic theories and my final decision for this way in Rostock (1955).

After war discussions between Jordan and Pauli, apart from fundamental questions, referred to numerical estimates of Jordan's geological predictions, particularly to the origin of the magnetic field of the Earth. Finally Pauli's negative position convinced Jordan of his presumably wrong way. He stopped his own 5-dimensional research and with him the corresponding work of his Hamburg relativity group (1961). It should not be forgotten that Jordan's concept of applying his new ideas to geology was also attacked by some geologists, partially in an unfair way.

Nevertheless, it should be emphasized that Jordan had considerably pushed forward the 5-dimensional projective relativity theory, though he could not present an acceptable physical interpretation.

In this historical context one should in any case remember the monograph by G. Ludwig [4] who presented Jordan's theory on the level of those days in a very abstract and profound manner but without adequate application. A main accent of this book is devoted to the spinor theory.

Under complicated political circumstances I had the chance to discuss with Jordan at the Meeting of the German Physical Society in Frankfurt (Main) 1965 and some years later at the International Conference on General Relativity and Gravitation in London the controversial subjects mentioned. At that time I had already been involved in the projective-relativistic research since about a decade. My mathematical approach and my physical field equations as well as the interpretation were different to Jordan's, but our ideas to apply 5-dimensionality to the Earth were conform. My 5-dimensional projective-relativistic research since half a century turned step by step to astrophysics and cosmology.

2.2. Revival of the Kaluza-Klein Approach and the Step to String and Brane Theories with Even Higher Dimensions

Since the international contact between the scientists in the first years after World War II was practically dead, an exchange of ideas did not exist, here above all with respect to the Kaluza-Klein approach. Thus parallel work with the same aim happened in different countries, namely dropping the normalization condition to get a new field function, as explained above, for the projective-relativistic approach by Jordan. These steps, essentially on the basis of Riemannian geometry, were done in France by V. Thiry (1948), supervised by A. Lichnerowicz (1965), in Sweden by C. V. Jonsson (1951) and in the Soviet Union by Yu. B. Rumer (1955).

These various after war approaches were aiming at the same goal, but with different mathematical formalisms and different physical contents. Soon after this revival a fully new idea arose, namely leaving the 5-dimensionality and passing over to geometrical spaces of higher dimensions. I remember the adventurous step by J. Rayski (1965) to six and later to seven dimensions, and after that the publications of Yu. S. Vladimirov and co-workers (1982 and earlier), P. S. Wesson (Induced matter theory) and co-workers (1999 and earlier) etc. In this context one should also remind of the contributions of E. Witten (1981). The published material partially shows tendencies to the elementary particle physics.

The papers and books of those days just mentioned, had mostly the orientation to classical unified field theories. Since the door to higher-dimensional spaces had been opened, it was invitational to investigate thinkable physics in very high dimensions. More than a decade ago this development with the orientations to the theory of elementary particles stabilized at about 10 or 11 dimensions, but with the fully new concepts of string theories, membrane theories, brane theories of various different types. Here I have to stop the sketch of very high-dimensional spaces, since the fields of research just mentioned developed rapidly to own special branches of research, first with extremely positive prognoses, now somewhat more critical.

Nevertheless, I should at least mention the wide fields of daily progress, concerning the intermediate region between General Relativity Theory and the large area of the elementary particle theories: gauge field theories, particularly of Yang-Mills type; Supersymmetry theories (Susy); Supergravity theories; Great Unification Theories (GUT); Higgs theories, etc.

2.3. Space-Time with Curvature and Torsion

Returning to 4-dimensionality, I should draw the attention to an interesting parallel development of research, well elaborated since some decades: The 4-dimensional space-time is maintained, but its geometry is endowed with curvature (Einstein's gravitation) and torsion, a property with some analogues to continuum mechanics. The source of such a program is the generalizations of the Riemann geometry to Cartan-type geometries (1922/24). The first step in this direction came from D. W. Sciama (1962) and colleagues. I remember many profound discussions at international GR-conferences. Since several decades this subject was investigated and elaborated in detail by F. Hehl and his co-workers [5].

Shortly sketching the main idea, one can characterize this direction of research as follows: As already pointed out above, the 4-dimensionality of the space-time is maintained, but now this continuum has the properties of curvature and torsion. This means that the generalization of the Einstein field equations has to go the way to extended Einstein-like field equations being now described: The left hand side of this generalized equation has to be constructed of Cartan-type geometrical quantities, and the source term on the right hand side has to contain the energy-momentum tensor of matter for the curvature and a further part for the torsion, which is physically connected with the angular momentum tensor of the matter (spin matter) considered.

2.4. Scalar-Tensor Theories

This excurse finishing, I have to make some annotations to the field of the scalar-tensor theories, since this type of theories is again extraordinarily alive. The reason for this revival grounds on the extremely rapid progress in high-precision measuring techniques (all above distance measurements by laser ranging in general, lunar laser ranging, laser interferometry, etc.). The physical subject of this up-to day research is the empirical answer to the question: Does the temporal change of the gravitational constant exist at all? If yes, how large is the measuring value? The decision of this query would be of fundamental importance for determining the field theoretical research in future. Depending on the result of this measuring value, whole branches of field theories could possibly be filtered out of further use.

Let me for a moment return to Jordan's discussion with Pauli mentioned above. From Jordan's monograph cited one learns: He understood that even in the case that Pauli's rejection of a real 5-dimensionality of our Universe has to be accepted, his basic restriction to 4-dimensionality (but with the additional scalar field) offers enough field theoretical freedom for application in astrophysics and perhaps cosmology. In deed, he did a lot of further investigation on the basis of the tensor-scalar combination: tensor part for gravitation and scalar part for perhaps new effects. Particularly I should mention his numerical estimates of his introduced parameters with respect to the Einstein effects (above all the perihelion motion of Mercury).

Surveying the classical field-theoretical part of Jordan's lifework (beside his famous quantum-theoretical contributions), one finds lots of ideas for application in geophysics, astrophysics and cosmology. Due to interrupted international contacts after the war Jordan's contributions to extended field theories remained practically unknown in Western Europe and America. This fact I had to experience in my discussions at conferences. Nearly always I opened this scientifically fruitful door to Jordan's suggestions by my contributions.

It seems to be historically remarkable that Jordan in his monograph quoted above devoted to the projective-relativistic field theory about 30 pages but to his (by a scalar field) extended theory inclusive application about 100 pages.

Concretely, without knowing the corresponding activities of Jordan, the physicists C. Brans and R. H. Dicke [6] constructed on the basis of a 4-dimensional space-time a generalized gravitational theory by using an additional scalar field without any reference to 5-dimensionality (1961). This world-wide as Brans-Dicke theory known scalar-tensor theory, very carefully elaborated by these authors, dominated the discussions on conferences for several decades, but came later in difficulties with respect to basic discrepancies to the perihelion motion of Mercury. Nevertheless, versions of this theory play nowadays again an important role for calculating the time-dependence of Newton's gravitational constant. My proposal: One should call this theoretical package "Jordan-Brans-Dicke theory".

The historical development of the field theoretical research after the General Relativity Theory (1915) is extremely complicated. I tried to give a short hopefully understandable review on the most important essential ideas and intellectual constructions. I had to be very sparingly in offering quotations. With respect to elder historical facts the reader may look in my textbook [7] and for later material in my monograph on 5-dimensional field theories [8].

3. Theoretical Basis of the Projective Unified Field Theory

In the following I present a historical annotation on the research of the Projective Unified Field Theory (short: PUFT) which I developed since 1955 in three stages, using as geometrical basis elements the above described 5-dimensional projectors in the 5-dimensional projective space.

First stage [7, 9]

Without any restrictions with respect to the occurring scalar field presenting the geometry of the 5-dimensional projective space; 5-dimensional field equations including the non-geometrized matter, called substrate, by the substrate energy projector as source term on the right hand side of the field equation; 5-dimensional balance equation as a mathematical consequence of the field equation (as in the 4-dimensional Einstein theory with great convincing power); 5-dimensional continuum mechanical equation of motion of the substrate; 5-dimensional mechanical equation of motion of a test particle; elaborating of an elegant vectorial projection formalism for projecting these 5-dimensional equations into the 4-dimensional space-time with the result of 4-dimensional field equations, equations of mechanical motion, conservation laws; application of the results to mechanical motion and cosmology. Occurrence of physical interpretational difficulties by second order derivatives of the scalar field (already slightly indicated in some mathematical structures of the Jordan theory). During the next years attempts with success to overcome the second order terms mentioned by a kind of conformal transformation.

Second stage [10, 11]

On the 9$^{\text{th}}$ International Conference on General Relativity and Gravitation (GR9 in Jena 1980) I gave a detailed review report on the state of research of PUFT: mathematical theory in five and four dimensions as well as my individual physical interpretation and philosophical view on the 5-dimensional projective space and the 4-dimensional space-time. More in detail in my monograph [8].

Here I should inform on following cosmological aspect: In my 5-dimensional projective field equation I already in those days introduced an additional "scalaric-cosmological term" of an immense cosmological importance, being maintained after the projection into space-time. Within this framework this term corresponds to the well-known "cosmological term" of Einstein, but it differs through variability from the latter.

Since nearly one decade one knows from empirical facts that about $(7 \text{ to } 8) \cdot 10^9$ years ago the cosmological expansion of our Universe changed from decelerated to accelerated expansion. Nowadays this effect is explained by acting of "dark energy", up-to now without an accepted theory, but with some success described by Einstein's cosmological term. As in detail shown later, the scalaric-cosmological term will already give a theoretical answer.

My physico-philosophical position to PUFT presented in detail at GR9 (1980) reads:

This theory, in the meantime fully elaborated and applied to terrestrial, planetary, astrophysical and cosmological subjects is, of course, a hypothesis which must be proved empirically. It is a closed self-contained thoughts-construction pointing to a series of interesting physical phenomena and effects, being extremely small in our present cosmological era and therefore nearly not measurable, but possibly nevertheless pointing to real 5-dimensional objectivity of our Universe: In the far past perhaps obvious transparent effects of adapted orders of magnitude, particularly not a Universe with a singular big bang, but with a regular "urstart" (in my terminology). In the far future a Universe with a determined "Cosmic Final Act" having been numerically calculated in PUFT, and not a whimpering, dead, cold chaos, as some authors suggested.

In the next sections I will show the numerical prognosis of PUFT. The received numerical results, compared to the measurements, show that a rather large probability speaks for some success of the hypothesis explained.

Now I would like to sketch my hypothetical physico-philosophical position on 5-dimensionality and 4-dimensionality:

PUFT is a semi-unified field theory, because the substrate (matter) was added ad hoc. In the following expositions I restrict to the geometrized theory only. The mathematical structures received by the projection multiply mentioned show acquaintance with a generalized gravitational theory and a generalized electromagnetic theory. But apart from these two subjects a further field equation occurs for the up-to now free scalar field mentioned above (Jordan, Thiry, Jonsson etc.). Later explication will show that the physical interpretation of this field equation makes it obvious to use instead of the scalar field the corresponding 'scalaric field'. This step is not a formal substitution but a physical step with very heavily weighting implications acting far into the region of language. It has to be done in order to avoid logical contradictions, as pointed out in several former publications.

In my basic hypothesis I take the phenomenon "scalarism" or in other context "scalarity", described by the scalaric field, on the same level of real physical existence as gravitation and electromagnetism. The quintessence of these explanations means that after projection into space-time three (in a certain sense equivalent) phenomena of Nature occur: gravitation, electromagnetism and scalarism.

Sometimes I am asked the question: We learned that the four physical dimensions mean three dimensions for space (Ortsraum) and one dimension for time. But what means the fifth dimension physically?

Answer: According to my geometrical picture of the relationship between the projective space and space-time, the direction of scalarity stands orthogonally to space-time. Therefore one could call the fifth dimension "scal", and the 5-dimensional projective space "space-time-scal".

Third stage [12]

In the preceding stage the complications with respect to the occurrence of the second order derivatives of the scalaric field could be avoided, but a small further change was performed because of some improvements in embedding the spinor theory into PUFT. Detailed information can be taken from [8].

4. Mathematical Explication of the Projective Unified Field Theory

A better understanding of the preceding theoretical treatise of the basic ideas, arguments, implications and interpretations of PUFT is reached by reading the following short sketch of the mathematical basis of this theory.

4.1. Field Equation in the 5-Dimensional Projective Space

My starting point was to test my hypothesis that we possibly are living in a 5-dimensional Universe governed by a real 5-dimensional physics. Therefore my primary goal meant constructing a 5-dimensional field equation within the 5-dimensional projective space with its own geometrical axiomatic, as simple as possible but as rich as necessary in order to cover that part of physics which can be grasped by the axiomatic of the 5-dimensional geometrical structure postulated. Then projecting this 5-dimensional field equation onto the 4-dimensional space-time in order to receive 4-dimensional field equations to be interpreted in an understandable adapted 4-dimensional language. As above pointed out, this projection procedure leads to three different areas of physics: gravitation, electromagnetism and scalarism. Now I would like to emphasize that the guideline for my following choice of the field equations was to investigate such field equations which for the specialization to the case of a constant scalaric field (σ = constant) should come as close as possible to the Einstein theory of gravitation and to the Maxwell theory of electromagnetism.

The postulated 5-dimensional Hamilton principle reads:

$$\delta W = \frac{1}{c}\delta \int\limits_{V_5} \overset{5}{L} \mathrm{d}^{(5)} f. \tag{2}$$

The quantities here occurring have the physical meaning: W is the action function, $\overset{5}{L}$ the Lagrange density, and $\mathrm{d}^{(5)}f$ a 5-dimensional volume element. The variation refers to the 5-dimensional metric tensor $g_{\mu\nu}$.

My Lagrange density being chosen is

$$\overset{5}{L} = -\frac{1}{2\varkappa_0 S}\left(\overset{5}{R} - \frac{4}{S^2} S_{,\alpha} S^{,\alpha} + \frac{2\lambda_S}{S^2}\right) + \overset{5}{L}{}^{(\Theta)}, \tag{3}$$

where $\overset{5}{L}{}^{(\Theta)}$ represents the substrate (non-geometrized matter).

This choice leads to the 5-dimensional field equation:

$$\begin{aligned} &R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \overset{5}{R} - \frac{1}{S} S^{,\mu;\nu} - \frac{2}{S^2} S^{,\mu} S^{,\nu} \\ &-\frac{4 s^\mu s^\nu}{S}\left(S^{,\tau}{}_{;\tau} - \frac{3}{2S} S_{,\tau} S^{,\tau} - \frac{S}{8}\overset{5}{R} - \frac{3\lambda_S}{4S}\right) \\ &+\frac{1}{S} g^{\mu\nu}\left(S^{,\tau}{}_{;\tau} - \frac{\lambda_S}{S}\right) = \varkappa_0 \Theta^{\mu\nu}. \end{aligned} \tag{4}$$

Here the curvature quantities are defined as usual:

$$\text{a)}\quad R_{\mu\nu} = R^{\alpha}_{\mu\nu\alpha}, \qquad \text{b)}\quad \overset{5}{R} = R^{\alpha}_{\alpha}. \tag{5}$$

Furthermore, following quantities occur: S is the amount of the 5-dimensional radius vector with the physical dimension of length ($S^2 = g_{\mu\nu} X^\mu X^\nu$), s^μ are the components of the unit vector in the direction of the radius vector, $\Theta^{\mu\nu}$ is the 5-dimensional energy projector of the substrate.

Let me further mention that in the field equation (4) three fundamental constants occur:

Einstein's gravitational constant, where γ_N is Newton's gravitational constant

$$\varkappa_0 = \frac{8\pi\gamma_N}{c^4}, \tag{6a}$$

the scalaric length constant of the same order of magnitude as Planck's length constant

$$S_0 = e_0\sqrt{\frac{\varkappa_0}{2\pi}} = 2.76 \cdot 10^{-34}\ \mathrm{cm}, \tag{6b}$$

and the dimensionless scalaric cosmological constant

$$\lambda_S. \tag{6c}$$

It proves convenient to use in the 4-dimensional space instead of S the scalaric field σ, defined by the equation

$$S = S_0 \mathrm{e}^{\sigma}. \tag{7}$$

For the following I would like to mention my conventions: Greek indices run from 1 to 5 (projective space), Latin indices from 1 to 4 (space-time). The signature is $(1,1,1,-1)$. In space-time the hat-index "4" is mostly suppressed, e. g. $\overset{4}{R} = R$.

Further it should be noticed that in this theory the analogue to the usual cylindricity condition is a characteristic Killing equation following from the axiomatic of the theory.

4.2. Field Equations in the Space-Time

As in astrophysics mostly applied, in this paper the Gauss system of units is used.

The projection mentioned leads to the following results.

Generalized Gravitational Field Equation

The projection of (2) leads to the 4-dimensional Hamilton principle

$$\delta W = \frac{1}{c}\delta \int\limits_{V_4} \overset{4}{L} \mathrm{d}^{(4)} f \tag{8}$$

with the relation for the 4-dimensional Lagrange density

$$\overset{4}{L} = \overset{5}{L} S. \tag{9}$$

Hence with the help of (3) results

$$\begin{aligned} \overset{4}{L} = -\frac{1}{2\varkappa_0}\Big(R + \frac{1}{4} B_{ij} H^{ij} + \frac{2}{S} S^{,i}{}_{;i} \\ -\frac{4}{S^2} S_{,i} S^{,i} + \frac{2\lambda_S}{S^2}\Big) + \overset{4}{L}{}^{(\Theta)} \end{aligned} \tag{10}$$

and further from (4) follows the 4-dimensional field equation

$$R^{mn} - \frac{1}{2}g^{mn}R - \frac{\lambda_S}{S_0^2}\mathrm{e}^{-2\sigma}g^{mn} = \varkappa_0(E^{mn} + S^{mn} + \Theta^{mn}). \tag{11}$$

Here R^{mn} and R are the usual 4-dimensional curvature quantities. Further the identifications are valid:

The energy-momentum tensor of the non-geometrized substrate

$$\Theta^{mn}, \tag{12a}$$

the electromagnetic energy-momentum tensor

$$E^{mn} = \frac{1}{4\pi}\Big(B^{mk}H^n_k + \frac{1}{4}g^{mn}B_{kl}H^{kl}\Big), \tag{12b}$$

and the scalaric energy-momentum tensor

$$S^{mn} = \frac{2}{\varkappa_0}\Big(\sigma^{,m}\sigma^{,n} - \frac{1}{2}g^{mn}\sigma_{,k}\sigma^{,k}\Big). \tag{12c}$$

The electromagnetic field strength tensor B_{ij} and electromagnetic induction tensor H_{ij} will be explained immediately in context with the electromagnetic field equations.

As already mentioned, the scalaric field σ is a new quantity in space-time. According to my hypothesis this field (with its origin in the 5-dimensionality) represents a new hypothetical phenomenon of Nature, parallel to the phenomena of gravitation and electromagnetism, which I called scalarism or scalarity, whose true existence as physical reality, of course has to be proved empirically in future.

From the last equations the usual gauge symmetry with respect to electromagnetism becomes obvious. Regarding the gauge symmetries of certain Yang-Mills theories, the situation here in the 5-dimensional concept is rather unique and also transparent.

In most applications for the description of the substrate the perfect energy-momentum tensor of a continuum is used:

$$\Theta^{mn} = -\Big(\mu + \frac{p}{c^2}\Big)u^m u^n - pg^{mn}, \tag{13}$$

u^m four-velocity, μ mass density, p pressure.

Generalized Electromagnetic Field Equations

The inhomogeneous electromagnetic equation

$$H^{mn}{}_{;n} = \frac{4\pi}{c}j^m, \tag{14a}$$

the cyclic electromagnetic equation

$$B_{<ij,k>} = 0, \tag{14b}$$

$$H^{mn} = \varepsilon B^{mn} \tag{14c}$$

with

$$\varepsilon = \mathrm{e}^{2\sigma}, \tag{14d}$$

the vacuum dielectricity/ polarisation. The quantity j^m means the electric current density, e. g. in the convective case: $j^m = \rho_0 u^m$, where ρ_0 is the rest charge density.

Scalaric Field Equation

$$\sigma^{,k}{}_{;k} - \frac{\lambda_S}{S_0^2}\mathrm{e}^{-2\sigma} = -\frac{\varkappa_0}{2}\Big(\frac{1}{8\pi}B_{ij}H^{ij} + \vartheta\Big) \tag{15}$$

with the definition of ϑ as difference of the traces of the substrate quantities shown:

$$\vartheta = \Theta_{\mu\nu}s^\mu s^\nu = \Theta^\mu_\mu - \Theta^m_m. \tag{16}$$

In this equation the scalaric substrate energy density (short: scalerg density) ϑ occurs as a basically new quantity within the framework of the traditional 4-dimensional physics. One of the main goals of treating the Klein-Gordon field in this paper is the intention to get arguments for a concrete choice of an appropriate formula for the scalerg density.

5. Balance Equations and Equations of Motions in the Space-Time

5.1. Balance Equations

Einstein's General Relativity Theory and the well-known gauge theories have, in contrast to some other field theories, the basic advantage that by mathematical operations the fundamental balance equations (particularly the related conservation laws of physics) result from the fundamental field equations, i. e. the balance equations mentioned are not independent axioms of his theory. Thus his theory exhibits a maximum of self-containment. Fortunately this advantage is also inherent in PUFT.

Here the mathematical operations mentioned above lead to following local balance equations:

$$\Theta^{mk}{}_{;k} = -\frac{1}{c}B^m_k j^k + \vartheta\sigma^{,m} \tag{17a}$$

for the energy momentum density,

$$j^m{}_{;m} = 0 \tag{17b}$$

for the electric current density. The second balance equation reflects the conservation of the electric charge.

5.2. Equations of motion

Inserting (13) into (17a) leads to the equation of motion of a mechanical continuum

$$\begin{aligned}&\left(\mu+\frac{p}{c^2}\right)u^m{}_{;k}u^k = \frac{\rho_0}{c}B^m_k u^k\\ &-\left(p^{,m}+\frac{1}{c^2}\frac{\mathrm{d}p}{\mathrm{d}\tau}u^m\right)-\vartheta\left(\sigma^{,m}+\frac{1}{c^2}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}u^m\right).\end{aligned} \tag{18}$$

The transition to a mechanical point-like body without internal degrees of freedom (e. g. rotation) gives the equation of motion

$$mu^m{}_{;k}u^k = \frac{Q}{c}B^m_k u^k - D\left(\sigma^{,m}+\frac{1}{c^2}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}u^m\right), \tag{19}$$

where the following volume integrals over the body were used:

The mass of the body

$$m = \int\left(\mu+\frac{p}{c^2}\right)\mathrm{d}^{(3)}V, \tag{20a}$$

the electric charge of the body

$$Q = \int \rho_0 \mathrm{d}^{(3)}V, \tag{20b}$$

the scalerg of the body

$$D = \int \vartheta \mathrm{d}^{(3)}V. \tag{20c}$$

Application of (19) to non-relativistic astrophysics, mostly considered under the approximations

$$\left(\frac{v}{c}\right)^2 \ll 1, \tag{21a}$$

$$\frac{|\phi|}{c^2} \ll 1, \quad \phi \text{ the gravitational potential,} \tag{21b}$$

leads to the useful non-relativistic equation of motion, where $\boldsymbol{v}$ is the velocity of the body, $\boldsymbol{E}$ the electric field strength, and $\boldsymbol{B}$ the magnetic field strength (induction):

$$\begin{aligned}&m\left(\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}+\boldsymbol{v}\frac{\mathrm{d}\ln\sigma}{\mathrm{d}t}+\operatorname{grad}\phi\right)=\\ &Q\left(\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}\cdot\boldsymbol{B}\right)-D\operatorname{grad}\sigma.\end{aligned} \tag{22}$$

6. Treatment of the Cosmological Model Using a New Relationship Between Scalerg Density, Mass Density and Pressure

6.1. Scalerg Density, System of Differential Equations, Time Dependence of the Mass of a Body Induced by the Expansion of the Cosmos

The cosmological model will be treated within the framework of PUFT. In spite of the comfortable use of numerical computer programs for solving the set of three coupled non-linear differential equations for three basic field quantities mentioned later, for mathematical reasons the model chosen will be rather simple: restriction to a 2-component gas mixture consisting of a substrate (matter) gas (at the beginning spinless dark matter particles with the property of later clumping at lower temperatures) and of an (electromagnetic) photon gas. In former papers I named the dark matter particles scalons.

As usual, the metric of such a cosmological model with the symmetry properties postulated (homogeneity, isotropy, spherical symmetry) reads:

$$\begin{aligned}\mathrm{d}s^2 = K(\xi)^2\left[\mathrm{d}\vartheta^2+\sin^2\vartheta(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2)\right]\\ -\mathrm{d}\xi^2,\end{aligned} \tag{23}$$

where K is the world radius and $\xi = ct$. Later the dimensionless time parameter $\eta = \frac{\xi}{A_0}$ including the rescaling factor $A_0 = 10^{27}$ cm is used for rescaling the three differential equations for the three quantities K, σ, μ with the parameter η, resulting from the above field equations (11) and (15). For the integration of this system, mostly done numerically, of course according to mathematical theorems I had to choose physically acceptable initial conditions, being similar to those of former papers.

In my textbook [8] and in my publication [13] I applied as an ansatz the rather simple relationship between the scalerg density and the mass density:

$$\vartheta = \frac{\mu c^2}{\sigma}. \tag{24}$$

The numerical results for the main cosmological parameters received for the empirical comparison to the results of measurements (age of the Universe, Hubble's expansion parameter, present mass density, etc.) were of the expected order of magnitude. Nevertheless, my further research showed that this outcome could be improved considerably by using instead of (24) the following relationship:

$$\vartheta = -(\mu c^2 + p)\mathrm{e}^{-2\sigma}. \tag{25}$$

How did I arrive at this formula with the surprising negative sign on the right hand side of this relation? As it is well known, the mass density μ and the pressure p are typical 4-dimensional physical quantities, whereas ϑ, according to (16), belongs originally to the 5-dimensional quantities, i. e. further information should come from the 5-dimensional trace Θ^{μ}_{μ}. Obviously the scalerg density is a bridge between the 5-dimensionality and the 4-dimensionality.

In the cosmological model to be investigated the scalaric field σ, which I named in this special case scalaric world function, only depends on time. Therefore a volume integration of (20c) can be performed. The result is the relation

$$D = -mc^2\mathrm{e}^{-2\sigma}. \tag{26}$$

This version came from the remembrance of my research on the 5-dimensional theories of the Klein-Gordon field and the Dirac field within the framework of PUFT, partially published several decades ago.

Let me now complete these considerations by following remark: Investigating a corresponding cosmological model with the metric (23) within the Einstein theory, H. Hönl [14] derived the Hönl relation

$$Km_0 \frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \text{const} \tag{27}$$

for a moving body in an expanding cosmos (m_0 constant mass of the body, v velocity of the body). A similar formula to (27) was derived by M. von Laue (1931) for the frequency of a moving photon in an expanding cosmos. I would like to emphasize that the well-tried relation (27) is in astrophysics and cosmology very important for the empirical test of new cosmological theories being proposed.

The next necessary step of integrating the basic field equations (11) and (15) now means to explicate this set of equations for the simplified cosmological model described above (homogeneity, isotropy, spherical symmetry and closeness of the cosmos) by using the metric (23). Rather lengthy calculations lead to the system of three differential equations:

$$\begin{aligned} &\frac{K''}{K} - \frac{2}{3}(\sigma')^2 - \frac{1}{3}\Lambda_{\mathrm{S}}\mathrm{e}^{-2\sigma} \\ &+ \frac{\varkappa_0}{6}(\mu c^2 + 3p) + \varkappa_0 p_{(r)} = 0, \end{aligned} \tag{28a}$$

$$\sigma'' + \frac{3K'\sigma'}{K} + \Lambda_{\mathrm{S}}\mathrm{e}^{-2\sigma} + \frac{\varkappa_0}{2}(\mu c^2 + p)\mathrm{e}^{-2\sigma} = 0, \tag{28b}$$

$$\begin{aligned} &\mu' + \left(\mu + \frac{p}{c^2}\right)\mathrm{e}^{-2\sigma}\sigma' + \frac{3K'}{K}\left(\mu + \frac{p}{c^2}\right) = 0, \\ &K' = \frac{\mathrm{d}K}{\mathrm{d}\xi}\,, \text{ etc.,} \end{aligned} \tag{28c}$$

where $p_{(r)}$ is the radiation pressure and $\Lambda_{\mathrm{S}} = \frac{\lambda_{\mathrm{S}}}{S_0^2}$ the length-dimensional scalaric-cosmological constant.

A consequence of this set is the intermediate differential equation

$$\begin{aligned} &\frac{1}{K^2}(K'^2 + 1) - \frac{1}{3}\Lambda_{\mathrm{S}}\mathrm{e}^{-2\sigma} + \frac{1}{3}(\sigma')^2 \\ &-\varkappa_0\left(\frac{1}{3}\mu c^2 + p_{(r)}\right) = 0. \end{aligned} \tag{29}$$

Treating this system of differential equations (28) and (29), I succeeded in arriving at an analogous relation to (27), however using a different scheme of definitions for the motion of a body. Since such a relationship was my intended goal, I had to postulate the following differential equation for the scalaric mass of the body considered:

$$\frac{\mathrm{d}m_\sigma}{\mathrm{d}\sigma} + m_\sigma(\sigma)\mathrm{e}^{-2\sigma} = 0. \tag{30}$$

The solution of this differential equation leads to the "cosmological scalaric mass formula" including the "iterated exponential function" [15]

$$m_\sigma(\sigma) = \frac{m_0}{\sqrt{\mathrm{e}}}\exp\left[\frac{1}{2}\exp(-2\sigma)\right] \tag{31}$$

with the initial condition $m_{\sigma 0} = m_\sigma(\sigma = 0)$ (constant of integration), where $\sigma = 0$ is the begin of time counting called in my terminology "urstart" as above introduced. One should realize that according to this formula via the scalaric world function the scalaric mass of a body changes with time in the course of expansion. Within this concept of PUFT this scalaric mass formula is rather stringent.

Returning to cosmology, the generalization of the Hönl relation (27) now reads:

$$Km_\sigma \frac{v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \text{const.} \tag{32}$$

Let me once more remind the reader that the urstart time point $t=0$, i. e. $\xi=0$ and $\eta=0$, designs the temporal beginning of the existence of the cosmos investigated. I called this regular begin "urstart", occurring instead of the unavoidable singular big bang ("urknall") in the Einstein theory. Here it seems not to be sensible to speculate on the question what was before the cosmos being under research.

This section concluding, I would like to emphasize that the predicted (temporal) change of the mass of a body, induced by the influence of the scalaric field function, is a new relevant outcome resulting from the hypothetical 5-dimensional physics. Further consequences for other regions of physics may occur. I remind of ideas being permanently discussed in present physical research literature: e. g. cosmological time dependence of the gravitational constant, of Sommerfeld's fine structure constant, of Planck's action constant, etc.

6.2. Equations of State, Rescaled System of Differential Equations, Initial Conditions

It was assumed that the cosmological gas in the closed cosmos may be a mixture of a photon gas with the radiation pressure $p_{(r)}$ and the perfect matter gas of scalons with the pressure p:

$$p_{(r)} = \frac{\Lambda_0}{K^4}, \tag{33a}$$

were Λ_0 is the radiation constant of the cosmos appearing as a constant of integration, and the perfect gas equation

$$p = nkT, \tag{33b}$$

where T is the kinetic temperature and n the particle number density of the perfect gas. Now a quasi equation of state (relationship between the mass density and the pressure) has to be chosen. I follow my former considerations [8], somewhat extended by the new insight into the temporal mass dependence of the scalaric field:

$$p = \frac{\mu c^2}{3(1+H_S)}. \tag{34}$$

Here the scalaric-cosmological pressure parameter is defined as follows

$$H_S = \left(\frac{m_\sigma c K}{\bar{C}_0}\right)^2, \tag{35a}$$

$$H_{S0} = \left(\frac{m_{\sigma 0} c K_0}{\bar{C}_0}\right)^2 \quad \text{for} \;\; \eta = 0, \tag{35b}$$

where the urstart parameter (constant of integration)

$$\bar{C}_0 = \frac{m_{\sigma 0} v_0 K_0}{\sqrt{1-\left(\frac{v_0}{c}\right)^2}}, \quad v_0 = v(\eta=0) \tag{36}$$

occurs.

The next step consists in using the rescaled quantities already mentioned above:

$$L(\eta) = \frac{K(\xi)}{A_0}, \tag{37a}$$

$$\eta = \frac{\xi}{A_0}, \qquad A_0 = 10^{27}\ \text{cm}. \tag{37b}$$

Eliminating both the pressures in the system of differential equations (28) and (29) by means of (34) and (33a) leads to the following rescaled system of differential equations:

$$\begin{aligned} &\ddot{L} - \frac{2}{3}L\dot{\sigma}^2 - \frac{1}{3}\Lambda_S A_0^2 \mathrm{e}^{-2\sigma} \\ &+ \frac{1}{3}\varkappa_0 c^2 A_0^2 \frac{1+\frac{H_S}{2}}{1+H_S} L\mu + \frac{\varkappa_0 \Lambda_0}{A_0^2 L^3} = 0, \end{aligned} \tag{38a}$$

$$\begin{aligned} &\ddot{\sigma} + \frac{3\dot{\sigma}\dot{L}}{L} + \Lambda_S A_0^2 \mathrm{e}^{-2\sigma} \\ &+ \frac{2}{3}\varkappa_0 c^2 A_0^2 \frac{1+\frac{3H_S}{4}}{1+H_S} \mathrm{e}^{-2\sigma}\mu = 0, \end{aligned} \tag{38b}$$

$$\begin{aligned} &\dot{\mu} + 4\mu \frac{1+\frac{3H_S}{4}}{1+H_S}\left(\frac{\dot{L}}{L} + \frac{1}{3}\mathrm{e}^{-2\sigma}\dot{\sigma}\right) = 0, \\ &\dot{L} = \frac{\mathrm{d}L}{\mathrm{d}\xi}, \text{ etc.,} \end{aligned} \tag{38c}$$

and

$$\begin{aligned} &\dot{L}^2 + 1 + \frac{1}{3}L^2\dot{\sigma}^2 - \frac{1}{3}\Lambda_S A_0^2 \mathrm{e}^{-2\sigma} L^2 - \frac{\varkappa_0 \Lambda_0}{A_0^2 L^2} \\ &- \frac{1}{3}\varkappa_0 c^2 A_0^2 \mu L^2 = 0. \end{aligned} \tag{39}$$

For the further calculations it is convenient to use instead of (35b) the scalaric-cosmological gas parameter

$$z_{\mathrm{S}} = \frac{H_{\mathrm{S}0}}{\mathrm{e}L_0^2} = \frac{\left(\frac{c}{v_0}\right)^2 - 1}{\mathrm{e}L_0^2}, \quad v_0 = v(\eta = 0). \quad (40)$$

The integration of differential equations means to know the cosmological initial conditions for the urstart at $\eta = 0$. Similarly to the formulation of the initial conditions in former publications [8] here the following choice is found by rather long probing (partly in Gauss units):

$$\begin{aligned} &\text{a)} \quad L_0 = 5 \cdot 10^{-5}, \quad \text{b)} \quad \sigma_0 = 0, \\ &\text{c)} \quad \dot{\sigma}_0 = 5.832 \cdot 10^8, \quad \text{d)} \quad \mu_0 = 8 \cdot 10^{-11}, \\ &\text{e)} \quad z_{\mathrm{S}} = 2.94746 \cdot 10^6, \quad \text{f)} \quad v_0 = 0.999, \\ &\text{g)} \quad \lambda_{\mathrm{S}} = 4.4 \cdot 10^{-122}. \end{aligned} \quad (41)$$

Cosmological experience led to the choice for the present era indicated by the index p:

$$\text{a)} \quad L_{\mathrm{p}} = 35, \qquad \text{b)} \quad \eta_{\mathrm{p}} = 13. \quad (42)$$

6.3. New Results for the Cosmological Parameters

Concluding this section I would like to inform the reader about the outcome: The usual numerical integration of the cosmological system of differential equation, supported by physically acceptable initial conditions at the cosmological urstart leads to following numerically satisfying results for the present cosmological era in good agreement with the empirical experience (y = year):

$$t_{\mathrm{aU}} = 13.74 \cdot 10^9 \text{ y}, \quad (43a)$$

age of the Universe,

$$K_{\mathrm{U}} = 3.5 \cdot 10^{28} \text{ cm}, \quad (43b)$$

curvature radius of the Universe,

$$H = 71 \frac{\text{km}}{\text{sMpc}}, \quad (43c)$$

Hubble's expansion parameter,

$$q = -1.02, \quad (43d)$$

deceleration parameter,

$$t_{\mathrm{acc/exp}} = 7.93 \cdot 10^9 \text{ y}, \quad (43e)$$

beginning of the accelerated expansion, caused by the scalaric-cosmological term,

$$\mu = 3.33 \cdot 10^{-30} \text{ g cm}^{-1}, \quad (43f)$$

mass density, primarily dark matter particles with the property of later clumping,

$$G_{\mathrm{S}} = \gamma_{\mathrm{N}} \left(1 - \mathrm{e}^{-4\sigma}\right), \quad (43g)$$

time-dependent scalaric-gravitational parameter instead of Newton's gravitational constant,

$$\frac{1}{G_{\mathrm{S}}} \frac{\mathrm{d}G_{\mathrm{S}}}{\mathrm{d}t} = -1.5 \cdot 10^{-13} \text{ y}^{-1}, \quad (43h)$$

relative temporal change of the scalaric-gravitational parameter.

Further results can be found in a new monograph [16].

7. 5-Dimensional Klein-Gordon-Field

7.1. General Remarks

Let me remind that PUFT is in the case of a vanishing substrate (vanishing energy-projector in the 5-dimensional projective space and therefore vanishing energy-momentum tensor as well as electric current density in the 4-dimensional space-time) a true geometrical unified field theory for the phenomena gravitation, electromagnetism and scalarism. If substrate is present, because of missing deeper knowledge up to now, PUFT is a semi-unified field theory. In order to receive urgently needed information on the phenomenon of the substance existing in our real world, here for the reason of modeling the 5-dimensional Klein-Gordon field including the coupling to the electromagnetic field is studied. Since even the sketch of the 5-dimensional Dirac field is rather voluminous, in my short report on the Dirac theory I abstain in this case from treating the electromagnetic coupling.

The 5-dimensional Klein-Gordon field and Dirac field were fragmentarily treated by several authors and finally in an extensive way by W. Pauli [2], but not on the post war level.

By the following consideration I try to get a justification for the ansatz (25).

7.2. Klein-Gordon Field

In section 6.1 the cosmological importance for taking equation (25) as a bridge between 5-dimensionality

(ϑ, σ) and 4-dimensionality (μ, p) has been explained. As pointed out there, the idea for this fruitful relation came from the 5-dimensional Klein-Gordon theory which now will be treated within the 5-dimensional framework of PUFT.

In order to save space I immediately start with a complex 5-dimensional Klein-Gordon field $\Phi(X^\mu)$ (X^μ 5-dimensional projective coordinates) related to an electrically charged particle. As usual the partial derivative is denoted by a comma, further a star means complex conjugation. In contrast to the real tensorial field functions with real degrees of homogeneity, I postulate the following imaginary degree of homogeneity for the complex Klein-Gordon field:

$$\Phi_{,\mu} X^\mu = i\alpha_S \Phi \tag{44a}$$

with

$$\Phi^*{}_{,\mu} X^\mu = -i\alpha_S \Phi^*. \tag{44b}$$

In my theoretical concept [11] the quantity

$$\alpha_S = \frac{e_0^2}{\hbar c} \tag{45}$$

is the dimensionless Sommerfeld fine structure constant and $\hbar$ the modified Planck constant.

In the Klein-Gordon theory the 4-dimensional complex Klein-Gordon field $\overset{4}{\Phi}(x^i)$ is not received by a vectorial projection, but by the following algebraic connection:

$$\Phi(X^\mu) = \overset{4}{\Phi}(x^i) F(X^\mu)^{i\alpha_S}, \tag{46}$$

where $F(X^\mu)$ is a real 5-dimensional function with the homogeneity degree of freedom 1:

$$F_{,\mu} X^\mu = F = F^*. \tag{47}$$

I was successful in finding a concrete real function with a geometric-physical meaning obeying the conditions demanded, namely

$$F = \left(\sqrt{\overset{5}{g}}\right)^{-\frac{1}{5}}. \tag{48}$$

The quantity $\overset{5}{g} = -\det(g_{\mu\nu})$ is the real 5-dimensional metrical determinant [17].

Let me first restrict my considerations to the 5-dimensional homogeneous coordinate transformations which according to Jordan in 4-dimensional space-time represent both, the coordinate transformations for tensors, e. g. for the electromagnetic potential

$$A_{m'} = \frac{\partial x^k}{\partial x^{m'}} A_k; \tag{49}$$

and its gauge transformation

$$\tilde{A}_m = A_m + \chi_{,m}, \quad \chi \text{ gauge function.} \tag{50}$$

Postulating the Klein-Gordon field as an invariant with respect to 5-dimensional coordinate transformations:

$$\Phi' = \Phi \tag{51}$$

leads to the relationship between the 4-dimensional and 5-dimensional derivatives of the Klein-Gordon field:

$$\Phi_{,\mu} g^\mu_k = \left(\overset{4}{\Phi}{}_{,k} - \frac{ie_0}{\hbar c} A_k \overset{4}{\Phi}\right) F^{i\alpha_S}, \tag{52}$$

where g^μ_k are the projection coefficients and

$$A_k = -e_0 (\ln F)_{,\mu} g^\mu_k = \frac{e_0}{5} \left(\ln \sqrt{\overset{5}{g}}\right)_{,\mu} g^\mu_k. \tag{53}$$

This last relation between the 5-dimensional metric and the electromagnetic potential represents the 5-dimensional geometrization with respect to the potential mentioned, whereas the first one explicitly shows the electromagnetic coupling.

Here one should remember that the second partial derivatives of invariants are no tensors. Therefore in the following the use of the well-known covariant derivative

$$\Phi_{,\mu;\nu} = \Phi_{,\mu,\nu} - \left\{ {\alpha \atop \mu\nu} \right\} \Phi_{,\alpha}, \qquad \left\{ {\alpha \atop \mu\nu} \right\} \text{ the Christoffel symbol,} \tag{54}$$

is necessary.

7.3. *Listing the Most Important Equations of the Linear Klein-Gordon Field*

Some relations I took from the corresponding scientific material [11, 17]. Here I would like to restrict to the case of a vanishing external electromagnetic field.

Lagrangeans:

$$\overset{5}{L}{}^{(\Theta)} = -\frac{\hbar^2}{2m_0 S}\left(\Phi^*_{,\mu}\Phi^{,\mu} + \frac{m_0^2 c^2}{\hbar^2}\Phi^*\Phi\right), \quad (55a)$$

$$\overset{4}{L}{}^{(\Theta)} = -\frac{\hbar^2}{2m_0}\left(\overset{4}{\Phi}{}^*_{,k}\,\overset{4}{\Phi}{}^{,k} + \frac{m_\sigma^2 c^2}{\hbar^2}\overset{4}{\Phi}{}^*\,\overset{4}{\Phi}\right) \quad (55b)$$

with

$$m_\sigma = m_0\sqrt{1 + \frac{\alpha_S^2\hbar^2 e^{-2\sigma}}{m_0^2 c^2 S_0^2}}, \quad (55c)$$

using a model particle, where m_0 is the constant rest mass and m_σ the specific time-dependent scalaric mass.

Field equations:

$$\Phi_{;\mu}^{;\mu} - \frac{1}{S}S_{,\mu} - \frac{m_0^2 c^2}{\hbar^2}\Phi = 0, \quad (56a)$$

$$\overset{4}{\Phi}{}_{;k}^{;k} - \frac{m_\sigma^2 c^2}{\hbar^2}\overset{4}{\Phi} = 0. \quad (56b)$$

The semicolon means the covariant derivative.

Substrate energy-projector:

$$\Theta_{\mu\nu} = -\frac{\hbar^2}{2m_0}\Big[\Phi^*_{,\mu}\Phi_{,\nu} + \Phi^*_{,\nu}\Phi_{,\mu} + (s_\mu s_\nu - g_{\mu\nu})\Big(\Phi^*_{,\alpha}\Phi^{,\alpha} + \frac{m_0^2 c^2}{\hbar^2}\Phi^*\Phi\Big)\Big]. \quad (57)$$

Substrate energy-momentum tensor:

$$\Theta_{mn} = -\frac{\hbar^2}{2m_0}\Big[\overset{4}{\Phi}{}^*_{,m}\,\overset{4}{\Phi}{}_{,n} + \overset{4}{\Phi}{}^*_{,n}\,\overset{4}{\Phi}{}_{,m} - g_{mn}\Big(\overset{4}{\Phi}{}^*_{,k}\,\overset{4}{\Phi}{}^{,k} + \frac{m_\sigma^2 c^2}{\hbar^2}\overset{4}{\Phi}{}^*\,\overset{4}{\Phi}\Big)\Big]. \quad (58)$$

Substrate current density:

$$j^m = -\frac{e_0\hbar}{2m_0 i}\Big(\overset{4}{\Phi}{}^*\,\overset{4}{\Phi}{}^{,m} - \overset{4}{\Phi}{}^{*,m}\,\overset{4}{\Phi}\Big). \quad (59)$$

Substrate scalerg density:

$$\vartheta = -\frac{\alpha_S^2\hbar^2 e^{-2\sigma}}{m_0 S_0^2}\overset{4}{\Phi}{}^*\,\overset{4}{\Phi}. \quad (60)$$

Just this result was the reason for my choice (25).

8. Dirac Field (Spinor and Bispinor Field)

There is no space to go into further detail on the spinor or bispinor calculus needed to describe the Dirac field which is a decisive represent for the study of the spinorial matter (substrate in my notation used above). In listing the main important results I follow the analogue of the Klein-Gordon field. I should mention here that in mathematical respect a considerable research on the spinor theory in five dimensions has been done by G. Ludwig [4].

For saving space instead of the spinor calculus I use the adequate but considerable shorter bispinor calculus. To avoid misunderstandings I should say that here the task is not to treat the problem of 5-dimensional bispinors as representations of the 5-dimensional coordinate transformation group, but to imbed the bispinors analogous to the Klein-Gordon field into the 5-dimensional projective space and to show by decomposition the physical content by passing over to the 4-dimensional space-time.

With respect to the algebraic basis of the spinor and bispinor mathematics within PUFT in context with space-time I would like to cite some own publications on this subject [18]. The detailed treatment is rather voluminous. Therefore I can here only present a short sketch to convey an impression. One should recognize that many authors prefer the abstract operator calculus, whereas in my research the 4-rows matrices dominate.

Denoting the bispinor field by $\Psi(X^\mu)$ and the corresponding adjugated (adjoint) bispinor field by $\overline{\Psi}(X^\mu)$ (adjugation means the complex conjugation and then multiplication by the transposition matrix) and using metrical bispinorial matrices γ^μ, which are general-relativistic 4-dimensional Dirac matrices with 4 rows, then the analogue to (44a) reads:

$$\Psi_{,\mu}X^\mu = i\alpha_S\Psi. \quad (61)$$

On this basis following results, without external electromagnetic field, are presented.

Lagrangeans:

$$\overset{5}{L}{}^{(\Theta)} = -\frac{\hbar c}{2S_0}e^{-\sigma}\Big(\overline{\Psi}\gamma^\mu\Psi_{;\mu} - \overline{\Psi}_{;\mu}\gamma^\mu\Psi + \frac{2m_0 c}{\hbar}\overline{\Psi}\Psi\Big), \quad (62a)$$

$$\overset{4}{L}{}^{(\Theta)} = -\frac{\hbar c}{2}\Big(\overline{\overset{4}{\Psi}}\gamma^k\,\overset{4}{\Psi}{}_{;k} - \overline{\overset{4}{\Psi}}{}_{;k}\gamma^k\,\overset{4}{\Psi} + \frac{2m_0 c}{\hbar}\overline{\overset{4}{\Psi}}\,\overset{4}{\Psi}\Big). \quad (62b)$$

The electromagnetic interaction is in these formulas omitted.

Field equations:

$$\gamma^{\mu}\Psi_{;\mu}+\frac{m_0 c}{\hbar}\Psi=0, \tag{63a}$$

$$\gamma^{n}\overset{4}{\Psi}{}_{;m}+\frac{m_0 c}{\hbar}\overset{4}{\Psi}=0. \tag{63b}$$

The derivation of the substrate energy-projector needs lengthy calculations using the Lagrange-Hamilton formalism with variations with respect to the metric. But in intermediate steps of calculation variations of the metrical bispinor-tensors with respect to the metric mentioned have to be used. Therefore the calculations of this quantity, being very complicated, are here omitted. It seems that for the case of the Dirac field the scalerg density vanishes: $\vartheta=0$.

Substrate energy-momentum tensor:

$$\begin{aligned}\Theta_{mn}=-\frac{\hbar c}{4}\Big[&\overline{\overset{4}{\Psi}}\left(\gamma_m\overset{4}{\Psi}{}_{;n}+\gamma_n\overset{4}{\Psi}{}_{;m}\right)\\&-\left(\overline{\overset{4}{\Psi}}{}_{;m}\gamma_n+\overline{\overset{4}{\Psi}}{}_{;n}\gamma_m\right)\overset{4}{\Psi}\Big].\end{aligned} \tag{64}$$

Since some further formulas (including electromagnetic coupling) for current densities are rather simple, they are listed in the following:

Electrical current density:

$$j_k=-\mathrm{i}e_0 c\overline{\overset{4}{\Psi}}\gamma_k\overset{4}{\Psi}. \tag{65}$$

This tensorial quantity fulfils the conservation law

$$j^k_{;k}=0, \tag{66}$$

whereas the pseudo-tensorial current density

$$J^k=\overline{\overset{4}{\Psi}}\gamma^k\gamma\overset{4}{\Psi} \tag{67}$$

satisfies the balance law

$$J^k_{;k}=\frac{2m_0 c}{\hbar}\overline{\overset{4}{\Psi}}\gamma\overset{4}{\Psi}. \tag{68}$$

The pseudo-tensorial quantity γ is defined by $\gamma_1\gamma_2\gamma_3\gamma_4$.

9. Some Conclusions

1. The observed accelerated expansion of our Universe (43e) can be explained as an immediate result of PUFT. The concept of dark energy is not needed (until now an accepted theory of dark energy does not exist).

2. The semi-unified PUFT for the case of treating a continuum as in cosmology has to be taken as a phenomenological theory describing the cosmos as a 2-component gas mixture, where the substrate (matter) gas uses basic notions of continuum mechanics like mass density, pressure, temperature, being reduced to notions like particles (named scalons), particle densities, etc. The decreasing velocity of these particles is regulated according to the analogue of the Hönl relation (32), not being explicated here in further detail. It seems to be sensible to interpret this gas as the empirically proved dark matter. Therefore I demanded for such spinless particles as a hypothesis the property of clumping at lower temperatures and thus of forming atoms, molecules, bodies, planets, stars and galaxies. Of course, this step is a purely phenomenological postulate. As it is well known, the research of the properties of clumping particles belongs to the theory of elementary particles, i. e. to quantum field theory.

3. Some perhaps acceptable considerations lead to the hypothesis that the "urmass" (premass) of the scalon is determined by the formula

$$m_{\mathrm{S}}=\frac{3k}{A_0 L_0 c^2}\left(\frac{3\Lambda_0}{a_{SB}}\right)^{\frac{1}{4}}, \tag{69}$$

where k is the Boltzmann constant, a_{SB} the Stefan-Boltzmann constant, $L_0=L(\eta=0)$ the initial condition of the rescaled world radius K, and Λ_0 the electromagnetic radiation constant of the cosmos. One should realize that this prediction is based on the phenomenological part of PUFT. The reader will find more details in my publications quoted. The numerical value of the urmass of a scalon reads:

$$m_{\mathrm{S}}=8.8\cdot10^{-31}\,\mathrm{g}=494\,\tfrac{eV}{c^2}. \tag{70}$$

This means that in contrast to present considerations which localize such particles in the high energy physics (e. g. Higgs particle) this mass (70) would fall into the low energy region if PUFT would prove to be an acceptable basis for this kind of cosmological research.

4. The problem of a time-dependent "gravitational constant" is explicitly solved by the corresponding formula (43h).

5. The Einstein effects (perihelion motion, frequency shift, deflection of light) remain mainly as in the Einstein theory (with extremely small corrections).

6. PUFT predicts heat production in moving bodies caused by the cosmological expansion and by the time-dependence of the scalaric world function [8].

7. Though there exists up till now no accepted general-relativistic quantum field theory, being an important final basis for any decisive progress in the theory of elementary particles, the automatic coupling of electromagnetism here may also be interesting for the electromagnetic coupling mechanism in the Kaluza-Klein theory, as pointed out by A. Macias and H. Dehnen [19], and in various types of Higgs theories.

8. Resuming all my experience of research in unified field theories of various dimensions I come to the conclusion that the resulting numerical values (43) and (70) as well as further predictions being not listed above lead me to the opinion that the probability for our real living in a 5-dimensional world (with very small effects in the present era of the history of our Universe and very large effects at its beginning) is rather high [16].

Acknowledgement

I am very grateful to Professor A. Gorbatsievich (University of Minsk, Belo-Russia) for scientific discussions and technical help. Further I have to thank Professor H. Dehnen (University of Konstanz) for his proposal to improve my manuscript by including the Hamilton-Lagrange formalism.

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